

# Exhibit 1

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## brief communications

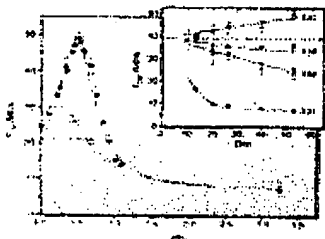
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*Nature* 414, 270 (2001); doi:10.1038/35104697**Brazil-nut effect: Size separation of granular particles**

Granular media differ from other materials in their response to stirring or jostling — unlike two-fluid systems, bi-disperse granular mixtures will separate according to particle size when shaken, with large particles rising, a phenomenon termed the 'Brazil-nut effect'<sup>1-5</sup>. Mounting evidence indicates that differences in particle density affect size separation in mixtures of granular particles<sup>2-11</sup>. We show here that this density dependence does not follow a steady trend but is non-monotonic and sensitive to background air pressure. Our results indicate that particle density and interstitial air must both be considered in size segregation.

Explanations of the Brazil-nut effect, which has been known since the 1930s, have focused either on infiltration of small particles into voids created underneath larger ones during shaking<sup>1-5</sup> or on granular convection<sup>6-8</sup>, and have implied density-independent rising times for the larger 'intruder' particles. However, an increase in the velocity of a large intruder with increasing density has been reported<sup>2,10</sup>, suggesting that increased inertia might play a role. Furthermore, in computer simulations<sup>10</sup>, a 'reverse' Brazil-nut effect was found, in which groups of larger particles, if heavy enough, segregate to the bottom.

A monotonic density dependence implied by such mechanisms<sup>2-11</sup> is incompatible with our measurements of intruder rising times over a wide range of size and density ratios (Fig. 1). We tracked an intruder particle in the presence of granular convection produced by vertically shaking a three-dimensional cylinder filled with smaller background particles (density,  $\rho_m$ ). A spherical intruder (diameter,  $D$ ; density,  $\rho$ ) was placed at a depth  $z_0$  below the surface; a hollow acrylic ball filled with foam and lead shot was used to tune the intruder density. Material properties other than density, such as coefficients of restitution and friction, had no measurable impact.

**Figure 1** Density and size dependence of the Brazil-nut effect.[Full legend](#)[High resolution image and legend \(42k\)](#)

For a fixed intruder diameter, the measured rising time,  $T_{\text{rise}}$ , to the free surface exhibits a pronounced peak as a function of  $\rho/\rho_m$  (Fig. 1). This peak is not affected by variations in shaking parameters, background medium (glass beads, poppy seeds) and system size. Compared with convection measured in the absence of an intruder (dotted line), the intruder rises faster both at large and small  $\rho/\rho_m$ , but more slowly when  $\rho/\rho_m \approx 0.5$ . A monotonic dependence,  $T_{\text{rise}} \sim (\rho/\rho_m)^{-1/2}$ , proposed for a two-dimensional system<sup>10</sup>, is incompatible with our data. The presence of a large intruder perturbs the convective flow of the background particles. Data above the horizontal dotted lines in Fig. 1 therefore do not necessarily imply sinking intruders<sup>2</sup> in the absence of convection. The peak in  $T_{\text{rise}}$  becomes significant for diameter ratios  $D/d > 10$ , increasing with increasing intruder size (Fig. 1, inset).

Measurements of intruder velocity as a function of depth show that the increase in  $T_{\text{rise}}$  with  $\rho/\rho_m$  to the left of the peak is caused by behaviour that takes place as the particle approaches the upper surface. Deeper inside the pile,  $T_{\text{rise}}$  decreases monotonically with  $\rho/\rho_m$ . The peak is sensitive to the background air pressure,  $P$ , in the cylinder. It decreases in magnitude and shifts to lower  $\rho/\rho_m$  with decreasing  $P$ , and vanishes as  $P$  approaches 1 torr. At this low pressure, the intruder velocity (both at the surface and within the bulk) no longer depends on  $\rho/\rho_m$  and co-incides, within our resolution, with the non-zero convection velocity of the background particles in the absence of the intruder.

Our results indicate an intricate interplay between vibration-induced convection and fluidization, drag by interstitial air<sup>12</sup>, and intruder motion. The rising time of a large intruder in a bed of smaller particles emerges as a sensitive probe of these interactions. Understanding the phenomenon described here may require a new approach that describes intruder motion in the presence of two 'fluids': background particles and interstitial air.

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## **Exhibit 2**

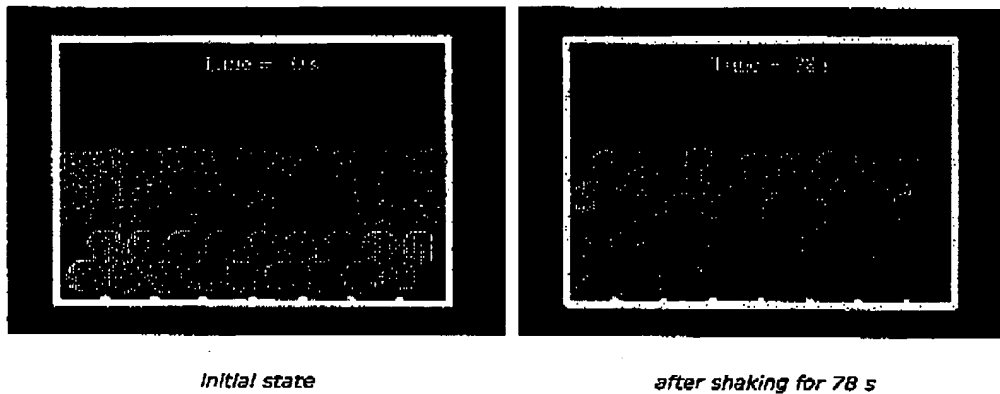
## Computational Fluid Dynamics

### Granular Flow - Application

#### Size segregation by a vibrating screen

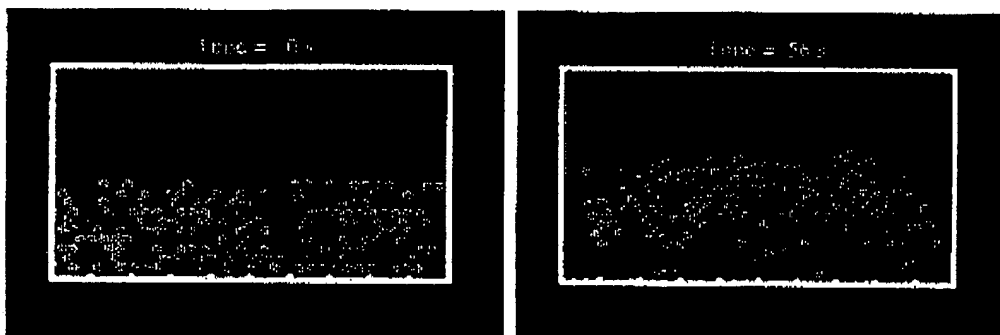
Contact personnel: P.W. Cleary

Separation of granular materials according to their physical properties is a very important process. Vibration induced segregation can be predicted by the DEM model. The figures below shows the initial state and state after 78 s of vibration of a box containing a binary mixture of 8 cm particles (Initially at the bottom) and 2 cm particles (Initially on top). The initial hexagonal microstructure of the large particles is very stable making this a good test.



As the shaking occurs, short lived gaps are created in the microstructure, allowing small particles to fall to lower levels. The intrusion of smaller particles prevents the larger particles from returning to their previous packed state. This allows further small particles to fall into the now long-lived gaps that have been opened between the large ones. Eventually the small particles reach the bottom. In this example, the small particles behave as invaders, slowly eating away at the microstructure of large particles. As each larger particle is separated it rises quickly to the surface. By  $t = 78$  s the large particles clearly occupy the upper part, demonstrating that size segregation can be predicted by DEM modeling. Further segregation is prevented by the large-scale convective motions generated by such vibration [1].

The figures below show the initial and final state of a binary mixture of 3 cm particles. The grey particles have a density of  $1600 \text{ kg/m}^3$ , while the pink particles are heavier with a density of  $3600 \text{ kg/m}^3$ . After shaking for 55 s, there is a clear concentration of heavy particles towards the bottom and lighter particles towards the top demonstrating that density segregation can be predicted by the DEM model. The mechanism responsible for density segregation is presently not well understood.



*initial state**after shaking for 55 s*

Rates of segregation can be predicted using the methodology described in [2] involving the calculation of the coefficient of variation of the distribution of local average diameter or density. This allows optimal vibrational modes to be identified that give the best separation rates for given types of materials.

Download animation: [AVI](#) (320x240 pixels; 19.8 MB)

[QuickTime](#) (320x240 pixels; 34.5MB)

#### References

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